New Thinking about College Mathematics

Implications for High School Teaching

Robin Marcus, Tim Fukawa-Connelly, Michael Conklin, and James T. Fey

NCTM's Standards and Navigations series, NSF-funded curricula, presentations at professional conferences and workshops, and countless articles in this journal offer many attractive ideas for introducing new mathematics, applications, and instructional approaches. After encountering such ideas, we invariably return to our mathematics classrooms with some great new lessons or enhancements to try. But unless the topics that pique our interest are on the high-stakes tests that our students face, we are inevitably stymied by the sense that we do not have time to cover essential concepts and skills *and* take even a couple of days off for mathematical explorations that are intriguing to students and teachers but are often considered not good use of classroom time by those responsible for political decisions. We have been puzzling over this frustrating situation—trying to reconcile the persuasive recommendations for change in the content and teaching of high school mathematics with the constraints of increasingly influential testing programs and prescriptive district curricula.

One salient development in recent thinking about high school mathematics is the notion that, for most students, secondary school should provide strong preparation for further mathematical work at the college level. In our work with teachers and district administrators, a commonly expressed argument against change is fear that students will not be prepared for college. When we investigated what the college mathematics community thinks about its curriculum, we found discussions and proposals that have significant implications for our own work. In particular, reports of the Mathematical Association of America (MAA) Curriculum Foundations Project raise important issues about the goals, content, and teaching of college preparatory mathematics. In this article, we provide highlights of the college recommendations, suggest implications for high school mathematics, and encourage readers to effect progressive change locally, on the school and district level, as well as on the state level.

What changes are afoot in college mathematics? How and why should those changes affect high school mathematics?

In 1999–2000, less than 8 percent of undergraduate students with declared majors were majoring in physics, engineering, or mathematics; more than 50 percent had chosen business, health, social and behavioral sciences, computer science, or life sciences—fields that have become highly mathematical (U.S. Department of Education 2000). From 1999 through 2001, the MAA held disciplinary workshops for representatives of these partner disciplines, including physics, engineering, and mathematics, to share what they want students to learn in the first two years of college mathematics. Each workshop generated a report, and a final summary conference led to the development of "A Collective Vision" (Ganter and Barker 2004a), a set of commonly shared recommendations for the first two years of undergraduate mathematics. "A Collective Vision" and the reports of each disciplinary workshop constitute *The Curriculum Foundations Project: Voices of the Partner Disciplines* (Ganter and Barker 2004b).

The project offers interesting and provocative ideas for those of us engaged in preparing students for the mathematical demands of college. "A Collective Vision" begins with broad suggestions to guide reform of the treatment of all topics in the first two years of college mathematics:

Emphasize conceptual understanding —"Focus on understanding broad concepts and ideas …" (p. 3).

Emphasize problem-solving skills—Develop students' ability to "apply a variety of approaches …, to apply familiar techniques in novel settings, and to devise multi-stage approaches in complex situations" (p. 3).

Emphasize mathematical modeling—Provide students with extensive experience in creating, solving, and interpreting mathematical models using analytic, graphic, numerical, and verbal reasoning and communication modes. "Students need to see mathematics in context" (p. 4).

Emphasize communication skills—"Incorporate development of reading, writing, speaking, and listening skills.… Require students to explain mathematical concepts and logical arguments in words. Require them to explain the meaning—the hows and whys—of their results" (p. 4).

These recommendations, while directed to college mathematics instructors, share a striking correspondence with NCTM's (2000) Process Standards for Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. They further echo the Learning Principle—"Learning mathematics with understanding is essential" (p. 20)—as well as the Curriculum Principle—"[T]he curriculum should offer experiences that allow students to see that mathematics has powerful uses in modeling and predicting real-world phenomena" (pp. 15–16).

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In addition to these general emphases, the Curriculum Foundations Project participants provide specific curriculum recommendations for the first two years of college mathematics. When considering the content of courses, the top six priorities outlined in "A Collective Vision" are the following (in order):

Aim for depth over breadth—"Topics can and should be eliminated to achieve a depth of conceptual understanding on a limited number of mathematical tools" (p. 5).

Incorporate statistics and data analysis—Motivate with "a variety of examples and real data sets, including data collected by students" (p. 5).

Give significant attention to discrete mathematics and mathematical reasoning—"Discrete mathematics … is absolutely essential for students majoring in computer science" (p. 5).

Refocus calculus and linear algebra—"[T]hese courses have often drifted in a theoretical direction that has made them obscure, formidable, and seemingly irrelevant to other disciplines.… Only the most fundamental and applicable results from calculus are needed" (pp. 5–6).

Replace college algebra—"De-emphasize intricate algebraic manipulations" and focus on "problem solving, mathematical modeling, descriptive statistics, and applications" (p. 6).

Develop multidimensional topics—Develop "vectors…, geometric and graphical reasoning, linear systems, and three-dimensional visualization skills" (p. 6).

These recommendations for the college mathematics curriculum resonate with and lend support to NCTM Standards (1989, 2000) for grades 9–12.

The discussion accompanying the advice to replace traditional college algebra courses asserts that neither "the overwhelming majority for whom this is a terminal course in mathematics, [nor] the relatively small minority for whom it is a gateway to further mathematics,… is well-served by the traditional version of the college algebra course" (p. 6). This suggests that modeling high school mathematics courses after or even preparing students for traditional college algebra courses is ill advised. If college professors argue that problem solving, data analysis, discrete mathematics, visualization, and mathematical modeling are substantially more important for students to learn than the mindless symbol pushing that traditionally dominates the precalculus curriculum in college, then it follows that students will be better prepared for college if high school mathematics curricula similarly shift emphasis.

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The need for increased emphasis on mathematical modeling was the strongest message from the Curriculum Foundations Project. Every disciplinary group in every survey workshop ascribed great importance to mathematical modeling. One might reasonably ask whether this has any relevance for high school curricula and teaching. We believe it does, and there are two principal justifications—one motivational, the other pedagogical.

First, national and international assessment reports show U.S. high school students performing at disappointing levels despite our best efforts to accelerate and intensify high school curricula (NAEP 2000; U.S. Department of Education 1998). Furthermore, nearly 30 percent of U.S. students drop out of high school before graduating (Green 2002). Blame for this disaffection with high school must, in large part, be attributed to a curriculum that is not meaningful or important to students. When our best defense of a topic or skill is "You'll need this next year for the SAT [or ACT, college placement, or other test]," we can expect all but the most committed students to give less than enthusiastic effort to learning. Any efforts to engage students in the realistic use of mathematics have to help. Our experience with problem-based curricula and teaching suggests that positive gains can be made in this direction (Schoen and Pritchett 1998).

The second convincing reason for introducing significant mathematical modeling work in high school is the too-little-appreciated fact that mathematical understandings like those required to analyze complex problem situations take time to develop (National Research Council 2000). Students cannot emerge from mathematical preparation that focuses on highly structured learning and practice of procedural skills and immediately apply that skill to genuine mathematical modeling tasks. One consistent and striking finding in evaluations of Standards-based middle and high school programs is that students who have extended experience with independent and collaborative work on complex and open-ended mathematical tasks become more capable, confident, and persistent problem solvers than those who have always worked in highly structured and guided learning environments (Senk and Thompson 2003; Stein and Lane 1996).

To achieve the mathematical content and process goals outlined in "A Collective Vision" and in NCTM's *Principles and Standards for School Mathematics* (2000), more is required than simply changing the content syllabi of our courses. If we are going to help students reach our broader goals of mathematical understanding, skills, and habits of mind, we probably need to teach in nontraditional ways. A number of individuals and projects are working on design and evaluation of such instructional alternatives. Drawing on ideas from many of those projects, "A Collective Vision" proposes some directions for innovation in college mathematics teaching:

Use a variety of teaching methods—Use more active alternatives to lecturing, "including in-class problem solving opportunities, class and group discussions, collaborative group work, and out-of-class projects" (p. 6).

Emphasize the use of appropriate technology—Technology can make previously inaccessible material approachable at early stages of students' mathematical development. "Technology is thus a powerful tool that should be utilized fully in the mathematics classroom"; however, "mathematics courses should stress intelligent and careful interpretation of results obtained from technology" (p. 7).

Use a variety of assessment strategies—Mathematics examinations must include items that assess conceptual understanding. "What you test is what you get" (p. 8).

These recommendations for a more student-centered instructional approach, appropriate use of technology, and varied assessment strategies reiterate the Principles and Standards set forth by NCTM (1991, 1995, 2000). Research provides supporting evidence that high school students experience greater learning gains when Standards-based curricular changes are bolstered by corresponding changes in instructional practice (McGaffrey et al. 2001).

For some time, we have felt that the most intriguing and potentially effective lever for changing the priorities of secondary school and college mathematics is the numerical, graphic, and symbol manipulation tools provided by calculators and computers. When asked to explain the computer skills and understandings that are fundamental to mathematical work in their fields, the Curriculum Foundations Project participants reported the importance of computing technology, particularly spreadsheets, to provide earlier access to complex and realistic problems and allow a focus on problem analysis and interpretation of results.

This aspect of technology and the mathematical modeling process can be illustrated in a wide variety of situations using different computing tools. When students create a spreadsheet to help them solve problems, model formulation and representation are clearly the most challenging tasks. The spreadsheet does the tedious calculation, leaving interpretation of results to the human problem solver. This benefit is mirrored by other software packages that perform sophisticated statistical tests, focusing attention on evaluating and choosing methods and then interpreting and communicating results. Similarly, a CAS executes complex symbolic manipulations, freeing users to concentrate on formulating algebraic models, selecting commands, and interpreting results.

To anyone who has been following or participating in the formulation and implementation of NCTM's recommendations for school mathematics, most of the points in "A Collective Vision" will sound unremarkable. Given the consistency of themes in the various recent reform recommendations, some may find puzzling how proposals for reform of mathematics curricula and teaching could be facing such difficulty when it comes time to implement the ideas. This disconnect between recommendations and reality brings us to the final big question of our analysis.

How can a progressive teacher or school system implement change in an era when school policy is driven by conservative high-stakes testing programs?

A company's profit (in thousands of dollars) from an item depends on the price of the item. Three different expressions for the profit at a price of p dollars follow:

$$-2p^2 + 24p - 54$$
$$-2(p - 3)(p - 9)$$
$$-2(p - 6)^2 + 18$$

How could you convince someone that the three expressions are equivalent?

Which form is most useful for finding—

(*a*) the break-even prices? What are those prices, and how do you know?
(*b*) the profit when the price is 0? What is that profit, and what does it tell about the business situation?
(*c*) the price that will yield maximum profit? What is that price, and how do you know?

Fig. 1 A sample problem that addresses *why* as well as *how* (adapted from McCallum, Connally, and Hughes-Hallett 2006)

Any individual or school, we believe, can move in directions that will prepare high school students for the kind of collegiate-level mathematical work required in a variety of disciplines. First, one of the most fundamental and important steps is to bring to our teaching—and explain or demonstrate to our students—a big-picture view of how mathematics helps in making sense of problematic situations. In even the most traditional skill-oriented high school course, one can show applications of the developing technical skills and frame discussion of those applications around principles of mathematical modeling.

The movement toward a curriculum that stresses *why* as much as *how* can begin in small ways: When a curriculum demands inclusion of items such as this—"Write the expression $-2p^2 + 24p - 54$ in factored and vertex forms"—one can also include items such as that in **figure 1**. A teacher could also take ideas for promoting discourse, student writing, and formative assessment and begin with an alternative treatment of standard topics.

These action suggestions are initial ways in which we can change our own classrooms. To accomplish more significant movement toward desirable goals, however, mathematics educators need to become more politically active, to make sure our voice is heard regarding important decisions about what and how we teach. We all have responsibility to become more assertive as advocates for the kind of mathematics curricula, teaching, and assessment that our professional judgment and research tell us will be best for our students.

Readers looking for support for moving in the directions recommended here have strong backing from the MAA and, of course, the NCTM. The priorities for the first two years of college mathematics outlined in "A Collective Vision" suggest that the future for which we are preparing students is going to be quite different from our past and from the current curriculum priorities and structures. We hope all readers share our excitement in exploring the new possibilities.

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ROBIN MARCUS, rmarcus@baltimorefreedomacademy.org, is a mathematics instructional coach at Baltimore Freedom Academy, Baltimore, MD 21231, and a doctoral student at the University of Maryland. TIM FUKAWA-CONNELLY, tim_fukawaconnelly@yahoo.com, is a high school mathematics teacher at Prospect Hill Academy Charter School, Cambridge, MA 02139, and a doctoral student at the University of Maryland. MIKE CONKLIN, conkmath@aol.com, is a mathematics teacher at University High School, Spokane Valley, WA 99216. JIM FEY, jimfey@umd.edu, is a professor of mathematics and mathematics education at the University of Maryland, College Park, MD 20742.